Office of Ratepayer Advocates

Value at Risk (VAR)

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Background of Risk Measurement in R.01-10-024

- ORA proposed that reasonableness be based on a rules based analytical system using either value at risk (VAR) or cash flow at risk (CFAR). (May 2002)
- ORA proposed that the utilities be required to use either VAR or CFAR to measure overall portfolio risk. (December 2002)
- D.02-12-074 (p.16) ordered that the Energy Division hold a workshop to gather information on VAR, CFAR, and other risk measurement models.

Current Situation

- SDG&E uses a VAR-to-expiration model.
- SCE uses a proprietary model.
- PG&E uses a proprietary model.

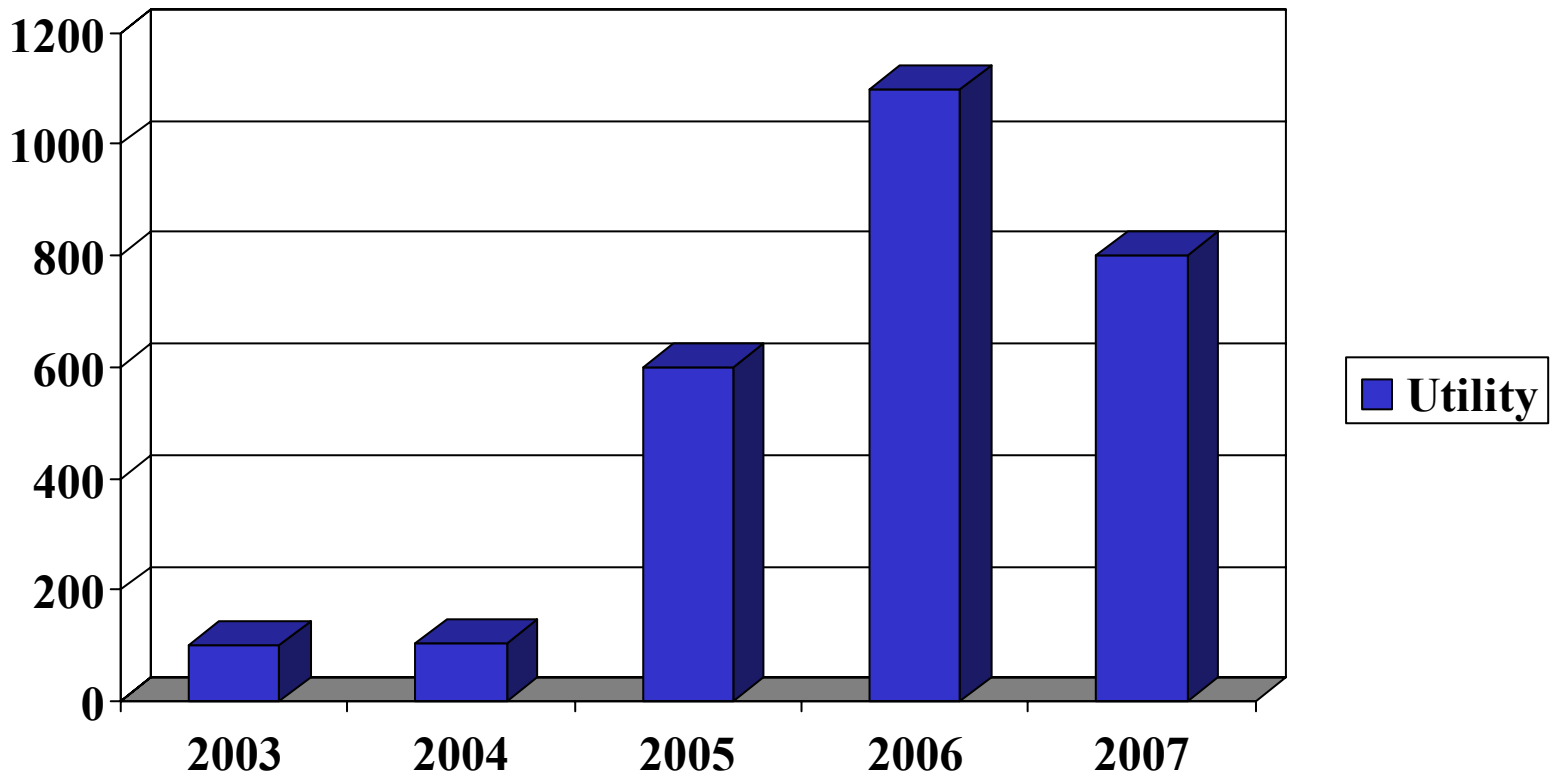
Risk Management History

- **1935:** Put and Call Brokers and Dealers Association was formed.
- **1973:** Black-Scholes Model was developed.
- **1973:** Chicago Board Options Exchange was formed.
- **1992:** Indiana Court of Appeals found that organizations have an obligation to hedge on behalf of their shareholders.
- **1994:** VAR model developed by J.P. Morgan

Risk Measurement Goals

- Standardization across Utilities
- Should be Probabilistic
- Not required to map to rates
- Provides Regulators and Utilities with a set of relative numbers that can be compared over time
- Allows Regulators and Utilities to be able to identify time periods when risk is projected to increase and to take appropriate action

Portfolio Risk Over Time



Linear VAR Model

- Assumes that changes in the value of the portfolio are linearly related to changes in the underlying market variables
- Assumes that changes in the underlying market variables are normally distributed
- Can be safely used when no options or fully dispatchable capacity contracts are part of the portfolio
- Forms the basis of the Quadratic VAR Model

Linear Formulas (95% confidence)

$$\sigma_{\Delta A} = Val_A Vol_A d^{0.5}$$

$$\sigma_{\Delta B} = Val_B Vol_B d^{0.5}$$

$$V_A = 1.65(\sigma_{\Delta A})$$

$$V_B = 1.65(\sigma_{\Delta B})$$

$$\sigma_{(A + B)} = (var_A + var_B + 2\rho\sigma_A\sigma_B)^{0.5}$$

$$V_P = 1.65[\sigma_{(A + B)}]$$

Linear Formulas (continued)

Where:

$\sigma_{\Delta A}$ and $\sigma_{\Delta B}$ are the standard deviations of the change in assets A and B.

Val_A and Val_B are the values of positions A and B in dollars.

Vol_A and Vol_B are the volatilities of assets A and B.

d is the number of days.

V_A and V_B is the VAR of positions A and B.

var_A and var_B are the variances of assets A and B.

ρ is the correlation coefficient between A and B.

V_P is the value at risk of the portfolio.

Estimating Correlation and Volatility

- **Option prices:** Calculate using traded option contracts.
- **Forward prices:** Use a time series of forward prices.
- **Spot prices:** Use a time series of spot prices.
- **Structural models:** A structural model (e.g., ARCH models) of the commodity market can be used to estimate correlation and volatility.

Creating Probability Distributions

When the actual probability distribution is non-normal (as is the case for most distributions) a representative distribution must be created using either:

1. Monte Carlo Simulation
2. Cornish-Fisher Expansion. The mean and standard deviation are taken from the Quadratic Model and are used in the Cornish-Fisher Expansion to create the third moment (skewness).
3. Hull-White Method. See "Value at Risk When Daily Changes are not Normally Distributed," *Journal of Derivatives*, Vol. 5, No. 3, pp. 9-19.

Monte Carlo Simulation

Monte Carlo Simulation can be used to obtain the probability distribution for ΔP (change in portfolio value). To calculate a one-day VAR:

1. Value the portfolio using the current values of market variables.
2. Sample once from the normal probability distribution of each variable (x_is) and use these values to determine the value of each market variable at the end of the day.
3. Revalue the portfolio at the end of the day
4. Subtract the value calculated in step 1 from the value calculated in step 3 to determine a sample ΔP.
5. Repeat steps 2 and 3 many times (e.g., 10,000) to build up a probability distribution for ΔP and build a distribution table.

Quadratic VAR Model

- Should be used when options are part of the portfolio.
- Accounts for Gamma (γ), the rate of change of delta with respect to the market variable.
- The probability distribution of ΔP is positively skewed when gamma is positive and negatively skewed when gamma is negative.
- A long call position typically has a positive gamma and a short call position typically has a negative gamma.
- Calculation of VAR is highly dependent on the left tail of the distribution.

Quadratic Formulas

Portfolio value is dependent solely on commodity S.

$$\Delta P = \delta \Delta S + 0.5(\gamma \Delta S^2)$$

Portfolio value is dependent on a number of variables (S_is)

$$\Delta P = \sum S_i \delta_i \Delta x_i + \sum 0.5[(S_i)^2(\gamma_i)^2 \Delta(x_i)^2$$

where Σ is the sum from i=1 to n, δ_i and γ_i are the delta and gamma with respect to the *ith* market variable.

Implementation

1. Initially, assume that RNS will be procured in the spot market.
2. Calculate VAR using a 5% probability.
3. Use the Quadratic model in combination with either the Cornish-Fisher expansion or Monte-Carlo simulation.

Definitions

Arbitrage is a zero-risk, zero-net investment strategy that still generates profits.

An **autoregressive model** is a forecasting model in which one uses the statistical properties of the past behavior of a variable to predict its behavior in the future. (i.e., $Y = c + b[Y(-1)]$)

The **Black-Scholes model** is primarily used to value stock options. The model uses the option strike price, forward price, and volatility of prices. The extended model assumes: (1) markets are efficient, (2) returns are lognormally distributed, and there are no arbitrage opportunities.

Definitions (continued)

A **Cap** is the maximum price to be paid for a commodity in a derivatives contract.

Cash Flow at Risk (CFAR) is a measure of uncertainty about future cash flows to a portfolio. CFAR gives the maximum shortfall within a specified confidence level (e.g., 95%) of the cashflows anticipated in a given planning period (e.g., July, 2003).

Collars specify both the upper and lower price limits for a commodity.

The **contract price** is the pre-determined price.

The **contract quantity** is the fixed quantity (e.g., 500 MW).

Definitions (continued)

The **correlation coefficient** is represented as a number between –1 and 1 which measures the degree to which two variables are linearly related. A correlation coefficient of 0 means that there is no linear relationship between the variables.

Correlation hedge is the purchase or sale of an asset that is highly correlated with another asset in a portfolio

The **delivery date** is the designated date in a forward or option contract.

Delta is the rate of change of the price of an option with respect to the price of the underlying asset.

Definitions (continued)

The **efficient market hypothesis** states that it is impossible to beat the market because prices already incorporate and reflect all relevant information.

The **exercise date** is the designated date in a forward or option contract.

A **floor** is the minimum price to be paid for a commodity in a forward or option contract.

A **forward curve** is a graph of estimated or actual forward prices over time.

A **forward long contract** is a contract to buy a commodity.

Definitions (continued)

A **forward short contract** is a contract to sell a commodity.

A **forward call option** is the right to buy an asset at a specific exercise price on or before a specific exercise date.

A **forward put option** is the right to sell an asset at a specific exercise price on or before a specific exercise date.

Gamma is the rate of change of the price of a portfolio of options with respect to the price of the underlying asset(s).

A **hedge is** a trade designed to reduce risk.

Definitions (continued)

Market risk is the uncertainty of future financial results due to changes in market rates and prices.

Monte Carlo simulation is a procedure for randomly sampling changes in market variables in order to value a contract or derivative.

An **option** gives the party the right to buy or sell a commodity at a future time for a specific price.

Rho is the rate of change of the value of a portfolio with respect to the interest rate.

Risk is the uncertainty of future prices.

Definitions (continued)

Risk Management is a set of strategies employed by firms and regulators to reduce the exposure of shareholders and ratepayers to uncertainty.

Skew is a measure of the distribution of a series around its mean.

Speculation is the act of investing with the intent of realizing a profit.

The **strike price** is the contract price. This is sometimes called the exercise price.

A **Swap agreement** is a contract that involves the exchange of physical commodities or cash flows according to a predetermined formula.

Definitions (continued)

Theta is the rate of change of the value of a portfolio as time passes. This is also referred to as the time decay of the portfolio.

The **underlying asset** is the asset on which a forward contract or option contract is written.

Value at Risk (VAR) is a measure of uncertainty about the future value of a portfolio. VAR measures the maximum loss within a specified confidence level (usually 95%) if the portfolio is held for a certain length of time.

Definitions (continued)

The **variance** of an asset's return is the expected value of the squared deviations from the expected return.

Vega is the rate of change of the value of a portfolio with respect to the volatility of the underlying asset(s).

Volatility is a measure of the uncertainty of the return realized on an asset. If volatility is to be expressed annually and monthly data is used, volatility is the standard deviation of returns multiplied by the square root of 12.